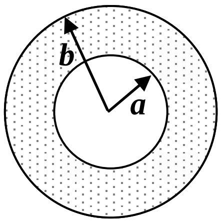
Pan Pearl River Delta Physics Olympiad 2010
2010 年泛珠三角及中华名校物理奥林匹克邀请赛
Part-1 (Total 6 Problems)　卷-1（共 6 题）
(9:00 am – 12:00 pm, 02-19-2010)

Math hints 数学提示: $\int x^n dx = \frac{1}{n+1}x^{n+1}$, $\int x\sqrt{A^2-x^2}\,dx = -\frac{1}{3}\left(A^2-x^2\right)^{\frac{3}{2}}$

Q1 Solve the following short problems. (11 points)
题 1 解下列各题。（11 分）

(i) Nucleus-A of mass M_A and kinetic energy E_0 ($<<M_Ac^2$) collides with nucleus-B (mass M_B) at rest. After collision nucleus-A moves at 90° relative to its initial velocity with kinetic energy E_1. Find the kinetic energy that has been converted to nuclear energy in the process. (Both nuclei can be viewed as free particles.) (2 points)
质量为 M_A 的原子核-A，初始动能 E_0 ($<<M_Ac^2$)，与初始静止的原子核-B（质量 M_B）碰撞。碰撞后原子核-A 以与初始速度成 90°的方向飞出，动能为 E_1。求在碰撞过程中由动能转变成核能的能量。（原子核可当作是自由粒子。）（2 分）

(ii) Find the resistance and the capacitance between two concentric spherical conductor shells of radii a and b ($> a$). The space between the shells is filled with medium with dielectric constant ε and conductivity σ. (2 points)
两同心圆壳导体，半径分别为 a 、b ($> a$)。两壳间充满介电常数为ε、导电率为σ的介质。求两导体之间的电阻和电容。（2 分）



(iii) (a) The electric field of an electromagnetic wave is $\vec{E} = E_0\vec{x}_0 e^{i(kz-\omega t)}$. What is its state of polarization? (1 point)
(b) Write down the expression in the same way as in (a) for the electric field of a left circularly polarized electromagnetic wave. (1 point)
(a)一电磁波的电场为 $\vec{E} = E_0\vec{x}_0 e^{i(kz-\omega t)}$ 。问电磁波的偏振状态是什么？（1 分）
(b)以与(a)相同的格式给出左旋圆偏振电磁波的电场。（1 分）

(iv) The electric field of an electromagnetic wave in vacuum is $\vec{E} = E_0\vec{x}_0 e^{i(kz-\omega t)}$. Find the magnetic field and the Poynting vector. (2 points)
真空中一电磁波的电场为 $\vec{E} = E_0\vec{x}_0 e^{i(kz-\omega t)}$ 。求电磁波的磁场和 Poynting 矢量。（2 分）

(v) Use the Lorentz transformation to derive the relativistic Doppler Effect when the relative motion between the source and the receiver is (a) parallel or (b) perpendicular to the wave propagation direction. (3 points)
利用洛伦兹变换，推出下列情形的相对论多普勒效应：(a)波的传播方向与波源、接收器的相对速度平行；(b) 波的传播方向与波源、接收器的相对速度垂直。（3 分）

Q2 (5 points) 题 2 （5 分）

(a) A spacecraft of mass m moves in a circular orbit at a distance r from the center of Earth (mass M_E). What is its kinetic energy? (1 point)
一质量为 m 的飞船，在离地球中心距离为 r 的圆形轨道上运行。地球质量为 M_E。求飞船的动能。（1 分）

(b) Suppose the spacecraft fires a rocket for a short time, speeding it up in the forward direction. After this short firing time, its kinetic energy increases by 30%. What is the furthest distance of the spacecraft from the center of Earth? Express your answer as a multiple of r. (4 points)
飞船启动火箭，在短时间内将飞船向前加速，使它的动能增加了 30%。问飞船离地球最远为多少 r？（4 分）

Q3 (9 points) 题 3 （9 分）

As shown below, a point light source is inside a glass block of refractive index n_G at distance y from the optical axis and distance T from the surface. A plano-convex lens made of transparent plastic material of refractive index n is placed near the source. The thickness of the lens is d, and the radius of its convex surface is R. The width of the air gap between the glass block and the lens is t.

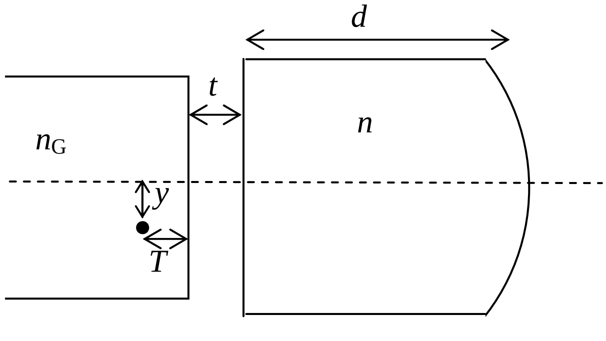
如下图所示，一点光源在玻璃内，与光轴的距离为 y，与玻璃平面的距离为 T。玻璃的折射率为 n_G。一透明塑料平凸透镜折射率为 n，厚度为 d，凸面的曲率半径为 R，与玻璃的距离为 t。

(a) Determine the width of the air gap t so that the light from the point source forms a parallel beam after passing through the lens. (3 points)
若点光源的光经过透镜后成为平行光束，求 t 的表达式。（3 分）

(b) Find the maximum thickness of the lens d_{max} such that it is still possible to adjust the width of the air gap to produce a parallel beam. (1 point)
当 d 大于 d_{max} 时，无论怎样调节 t 都无法得到平行光束，求 d_{max} 的表达式。（1 分）

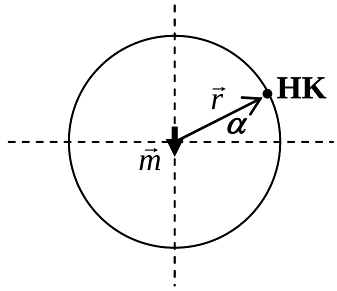
(c) Find the angle θ between the parallel beam and the optical axis. (3 points)
求平行光束与光轴的夹角θ。（3 分）

(d) If d is allowed to change by a small amount δd, and R is allowed to change by δR, find the angular change $\delta\theta$. (2 points)
若 d 改变一小值δd，R 改变一小值δR，求夹角的变化$\delta\theta$。（2 分）



Q4 (5 points) 题 4 （5 分）

The Earth's magnetic field is approximately a dipole field. One can imagine there is a magnetic dipole $\vec{m}$ at the center of the Earth. For simplicity, assume that $\vec{m}$ lies on the rotation axis of the Earth, pointing from north to south, as shown in the figure. Hong Kong is at the latitude of $\alpha = 22°$. The magnetic field at distance $\vec{r}$ from a dipole is $\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right]$.



如图所示，地球的磁场可当作为一个磁偶极子的场，偶极子 $\vec{m}$ 在地球中心，与地球自转轴同轴，由北指向南。香港在北纬 $\alpha = 22°$。离磁偶极子 $\vec{r}$ 处的磁场为 $\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right]$。

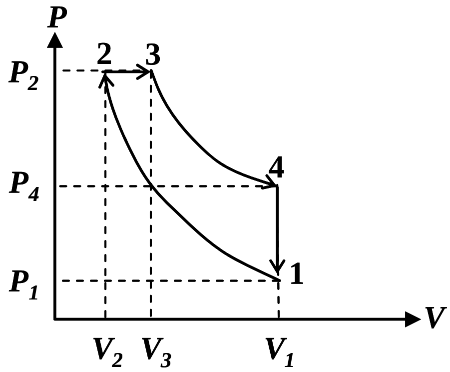
(a) Find the direction of the magnetic field in Hong Kong in terms of east, west, north, south, and the angle to the horizontal plain. (2 points)
求在香港的地球磁场方向（以东、南、西、北和水平面为基准）。（2 分）

(b) A horizontal electric wire of 10 meters in length carrying 100 A of current is in the north-south direction. Find the magnetic force on the wire. (You need to recall roughly the strength of Earth magnetic field on Earth surface.) (3 points)
一段南北走向的 10 米长的电缆，载有 100A 的电流。求地球磁场对电缆的力。（你需自己给出地球磁场的大约强度。）（3 分）

Q5 (8 points) 题 5 （8 分）

Shown in the figure is the Diesel ideal-gas cycle. All processes are quasi-static and the gas is monatomic. The two processes 1→2 and 3→4 are both adiabatic. Find the efficiency of the cycle in terms of $\alpha = V_3/V_2$ and $r = V_1/V_2$.

右图所示的是一个单原子理想气体的 Diesel 循环。循环中的每个过程都是准稳态的。1→2、3→4 为绝热过程。求循环的效率（用 $\alpha = V_3/V_2$ 、 $r = V_1/V_2$ 来表达）。

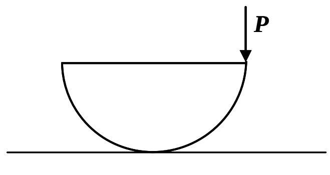


Q6 (12 points) 题 6 （12 分）

A uniform half disk of finite thickness with radius *R* and mass *M* is resting on a rough horizontal floor.

(a) Find the moment of inertia relative to the axis perpendicular to the half disk and through the contact point on the floor. (6 points)

(b) An impulse *P* is suddenly applied to the edge of the half disk as shown. The half disk rolls without slipping afterwards and maintains contact with the floor. Find the minimum impulse to flip the half disk. (6 points)



一个有一定厚度的均匀半圆盘，半径 *R*，质量 *M*，放在粗糙水平地面上。

(a) 一轴线与盘面垂直，并穿过半圆盘在平衡时与地面的接触点。求半圆盘相对于该轴线的转动惯量。（6 分）

(b) 一冲量 *P* 突然作用于盘的边缘。之后半圆盘作纯滚动，并一直保持与地面接触。求能使半圆盘翻转的最小冲量。（6 分）

《THE END 完》

Pan Pearl River Delta Physics Olympiad 2010
2010 年泛珠三角及中华名校物理奥林匹克邀请赛
Part-2 (Total 3 Problems) 卷-2（共 3 题）
(2:30 pm – 5:30 pm, 02-19-2010)

Math hints 数学提示: $\int x^n dx = \frac{1}{n+1}x^{n+1}$ **;** $\sum_{n=0}^{N} a^n = \frac{a^{N+1}-1}{a-1},\ N>1,\ a\neq 1$**.**

Q1 Chain Reactions (10 points)
题 1 连锁反应 （10 分）

(a) The mass of a proton is roughly equal to that of a neutron and is 1.67×10^{-27} Kg. Find the number of nuclei in 1000 Kg of ^{235}U. (3 points)
一个质子的质量和一个中子的质量相若，均为 1.67×10^{-27} Kg。求 1000 公斤 ^{235}U 所含的原子核数目。（3 分）

(b) When a ^{235}U nucleus is hit by a neutron, it captures the neutron, splits into several fragments, and releases 3 neutrons. These neutrons will hit 3 ^{235}U to create 9 neutrons, … The time between a neutron is released to the time it hits the next ^{235}U is 1.0×10^{-8} s. Estimate the total time needed to consume 1000 Kg of ^{235}U. (7 points)
当一个中子撞到一个 ^{235}U 原子核时，原子核会将中子俘获，然后裂变成数个碎块和 3 个中子。那 3 个中子分别撞向 3 个 ^{235}U 原子核，产生 9 个中子，… 中子从产生到撞到下一个 ^{235}U 原子核的时间间隔为 1.0×10^{-8} s。估计将 1000 公斤 ^{235}U 裂变完所需的时间。（7 分）

Q2 Quantum Gravity (20 points)
题 2 量子引力学（20 分）

(a) <u>The Planck Length</u>

The smallest length scale one can resolve is called the Planck Length L_P, which can be estimated in the following way. To get information within the length scale *L*, one needs a photon with the wavelength λ roughly the same as the length scale. But any photon creates a (tiny) blackhole within which no information can get out. When the photon wavelength is equal to 2π times the radius of the blackhole (=L_P), then we obtain the smallest length scale L_P. Note that a photon with wavelength λ has energy $E = hc/\lambda$ and effective mass $m = E/c^2$. (Planck Constant $h = 6.63\times10^{-34}\ J\cdot s$, speed of light in vacuum $c = 3.0\times10^{8}\ m/s$, universal gravity constant $G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$)

(i) Determine the Planck Length. (4 points)

(ii) Determine the Planck Energy E_P which is the energy of a photon with its wavelength equal to 2π times the Planck Length, in terms of electron volt (eV). You may find the constant $hc = 1240\ eV\cdot nm$ useful. ($1\ nm = 10^{-9}\ m$) (2 point)

(b) <u>The Hubble's Law</u>

(i) The light spectra of a distant galaxy exhibit a red shift of $Z \equiv (\lambda' - \lambda)/\lambda = 1.03$, where λ is the wavelength if the source is at rest relative to us, and λ' is the observed wavelength. Find the speed of the galaxy relative to us, and determine whether the galaxy is moving away or towards us. (7 points)

(ii) According to Hubble's Law, the receding speed of a galaxy *v* is proportional to its distance *D* from us, i. e., $v = H_0 D$, where H_0 = 21.7 (km/s)/Mly, and Mly represents a million light years. Find the distance of the galaxy. (2 point)

(c) Change of Speed of Light

Several quantum gravity models predict that photons with higher energies move slower, i .e., the speed of photon in vacuum with energy E is $v = c(1 - E/E_P)$, where E_P is the Planck Energy in (a). It can be tested by observing the light pulses from gamma-ray bursts, where a low energy photon pulse and a high energy photon pulse are emitted simultaneuously. A gamma ray burst was observed recently in the galaxy in (b). A low energy (~10^3 eV) photon pulse and a high energy photon pulse with energy of 34 × 10^9 eV was observed.

(i) According to the models, what would be the time delay between the two pulses when they reached us? (4 points)

(ii) The actual delay observed was 0.9 × 10^{-3} s. Does the evidence support the models' prediction? (1 point)

(a) 普朗克长度

我们可分辨的最短长度称为普朗克长度 L_P。下面我们来估计它的值。要获得长度为 L 的空间内的信息，我们需要的光子的波长λ要和 L 差不多短。但光子也有（很微小的）黑洞，黑洞内的信息在外面是无法得到的。令光子的波长等于 2π 乘以黑洞的半径(=L_P)，我们就得到最小的长度。已知光子的能量为 $E = hc/\lambda$，有效质量为 $m = E/c^2$。（普朗克常数 $h = 6.63\times10^{-34}\,J\cdot s$，真空光速 $c = 3.0\times10^8\,m/s$，引力常数 $G = 6.67\times10^{-11}\,N\cdot m^2/kg^2$）

(i) 求普朗克长度 。（4 分）

(ii) 求波长等于 2π 乘以普朗克长度的光子的能量，并将结果用电子伏特(eV)作单位。这便是普朗克能量 E_P。下面的常数你会觉得有用。（$hc = 1240\ eV\cdot nm$，$1\ nm = 10^{-9}\,m$）（2 分）

(b) 哈勃定理

(i) 某一个遥远的星系的光谱的红移为 $Z \equiv (\lambda' - \lambda)/\lambda = 1.03$。其中$\lambda$为静止光源的波长，$\lambda'$为我们实际测到的波长。求星系相对我们的速率，并确定星系是离我们而去，还是向我们飞来。（7 分）

(ii) 根据哈勃定理，星系飞离我们的速率 v 与离我们的距离 D 成正比，即 $v = H_0 D$，其中 H_0 = 21.7 (km/s)/Mly，Mly 为一百万光年。求星系离我们的距离。（2 分）

(c) 光速的变化

目前有些量子引力学的理论预测高能量的光子的速度会慢一点，即真空中能量为 E 的光子的速度为 $v = c(1 - E/E_P)$，其中 E_P 为(a)中的普朗克能量。要验证这一预测，我们需要观测伽玛射线的爆发。伽玛射线爆发时，光源会同时射出一个低能量和一个高能量光子的脉冲。最近，在(b)的星系我们观测到一次伽玛射线爆发。我们接收到一个低能量(~10^3 eV)光子的脉冲，和一个能量为 34 × 10^9 eV 的高能量光子脉冲。

(i) 根据理论预测，这两个脉冲到达地球的时间差是多少？（4 分）

(ii) 实际观测到的时间差为 0.9 × 10^{-3} s。这结果支持那些量子引力学的理论吗？（1 分）

Q3 The Sound of Bubbles (20 points)

题 3 气泡之声（20 分）

Consider an air bubble of radius R submerged in water of mass density ρ. In equilibrium the pressure inside and outside the bubble is P_0, if surface tension of water is ignored for the time being. Air can be treated as diatomic ideal gas. Given a small disturbance the bubble then vibrates radially at a characteristic harmonic frequency. The following steps can help you to find the frequency. The approach is to express the potential energy of the vibrating bubble in terms of Ax^2, and the kinetic energy in terms of $B\left(\frac{dx}{dt}\right)^2$, where x is the small displacement of the radius. The frequency is then $\omega = \sqrt{A/B}$.

(a) Suppose the radius of a bubble is changed adiabatically from R to $R + x$, ($x << R$), find the restoring force acting on the bubble surface which is proportional to x. (2 points)

(b) Find the energy required for x to change from 0 to a small value x_0. (2 points)

(c) The radial velocity of water at the bubble surface of radius R is $\frac{dx}{dt}$. Given that water is incompressible, what is the radial velocity of water at distance r (> R) from the bubble center? (3 points)

(d) Find the total kinetic energy of water. (2 points)

(e) The kinetic energy in (d) is that of the vibrating bubble, because the mass of air can be ignored. Find the vibration frequency. (1 point)

Now we consider the effect of surface tension of water on the vibration frequency. The surface tension comes from the fact that it takes energy γS to create a water surface of area S, where γ is the surface tension coefficient.

(f) Find the surface tension force on one of the four sides of a flat and square water membrane of side length a. (2 points)

(g) Find the surface tension energy needed to create an air bubble of radius R in water. (1 point)

(h) Find the pressure difference between the inside and the outside of the bubble in equilibrium. (2 points)

(i) Modify the answer to (e) to include the effect of surface tension of water. (5 points)

一半径为 R 的空气泡浸没在质量密度为ρ的水里。若暂时不考虑水的表面张力，则平衡时泡内、外的压强相同。设泡外压强为 P_0。空气可当作是双原子理想气体。因一小扰动，气泡开始以一特征频率作径向简谐振动。以下步骤能帮助你求出振动频率。其主要方法是将气泡振动的势能表达成 Ax^2，将动能表达成 $B\left(\frac{dx}{dt}\right)^2$，其中 x 为半径的微小位移量，然后可得特征频率 $\omega = \sqrt{A/B}$。

(a) 令气泡的半径由 R 绝热地变为 $R + x$, ($x << R$)，求作用于气泡面上的恢复力。该力应与 x 成正比。（2 分）

(b) 求将 x 从 0 变到 x_0 所需的能量。（2 分）

(c) 在半径为 R 的气泡表面，水的径向速率为 $\frac{dx}{dt}$。已知水是不可压缩的，求在离气泡中心 r (> R)处水的径向速率。（3 分）

(d) 求水的总动能。（2 分）

(e) 因空气的质量可忽略，(d)中的动能就是气泡振动的动能。求特征频率。（1 分）

现在让我们考虑水的表面张力对振动频率的影响。产生一面积为 S 的液体表面需要用能量γS，其中γ为表面张力系数。这就是表面张力的来源。

(f) 一四方平面液体薄膜边长为 a，求其中一边所受的表面张力。（2 分）

(g) 求水中一半径为 R 的气泡所含的表面张力能量。（1 分）

(h) 求该气泡在平衡时内、外压强之差。（2 分）

(i) 求包括水的表面张力效应的气泡特征振动频率。（5 分）

《THE END 完》